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In this issue:

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A Probability Model for Belady's Anomaly

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Abstract

In demand paging virtual memory systems, the page fault rate of a process varies with the number of memory frames allocated to the process. When an increase in the number of allocated frames leads to an increase in the number of page faults, *Belady's anomaly* is said to occur. In this paper, we present a probability model for Belady’s anomaly. We describe the use of computer simulation to estimate the parameters of the model over a design region of process sizes and reference string lengths. We then relate our probability model to the occurrence rate of Belady’s anomaly in the simulation results.

Keywords: Belady's anomaly, virtual memory, demand paging, page replacement, FIFO, Random Page, probability model, simulation.

1. INTRODUCTION

In Operating Systems courses, instructors often like to assign software projects that relate to course objectives. One topic with special appeal to students is Belady's anomaly. For a virtual memory system with demand paging, the page fault rate of a process varies with the number of memory frames allocated to the process. When an increase in the number of allocated frames leads to an increase in the number of page faults (a *bump*), *Belady's anomaly* is said to occur (Belady, 1969).

The occurrence rate for Belady's anomaly depends on which page replacement algorithm is implemented. Examples of Belady's anomaly for first-in-first-out (FIFO) paging systems are presented in commonly used Operating Systems textbooks (Silberschatz, 2008; Stuart, 2008; Deitel, 2004; etc.). However, Belady's anomaly cannot occur when page replacement is based on a stack algorithm (Mattson, 1970).

In an earlier study (McMaster, Sambasivam, and Anderson, 2009), we used computer simulation to describe conditions that affect *how often* Belady's anomaly will occur for the FIFO and Random Page algorithms. However, we did not explain *why* the observed patterns occur.

In the previous study, for each process size and reference string length in the design region, we counted the total number of page fault bumps generated by 1000 reference strings. Figure 1 summarizes the number of anomaly bumps for the FIFO algorithm.

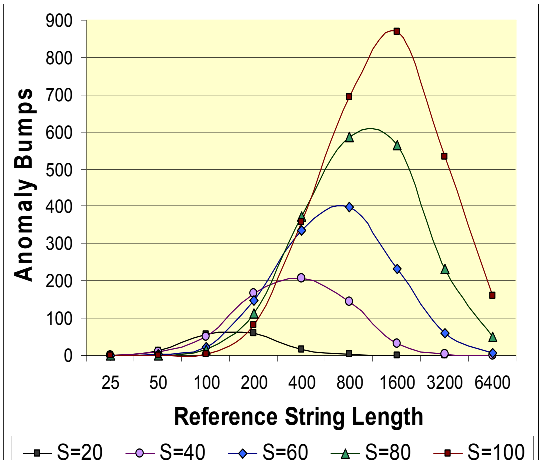

Figure 1. FIFO Anomaly Bumps per 1000 Reference Strings.

Anomaly bumps under FIFO appeared as often as 869 times per 1000 reference strings, with some strings having more than one bump.

The FIFO anomaly data used to generate Figure 1 is presented in tabular form as Table A1 in the Appendix.

Figure 2 presents the anomaly bump counts for the Random Page algorithm. Anomaly bumps under this algorithm occurred as often as 32152 times in 1000 reference strings.

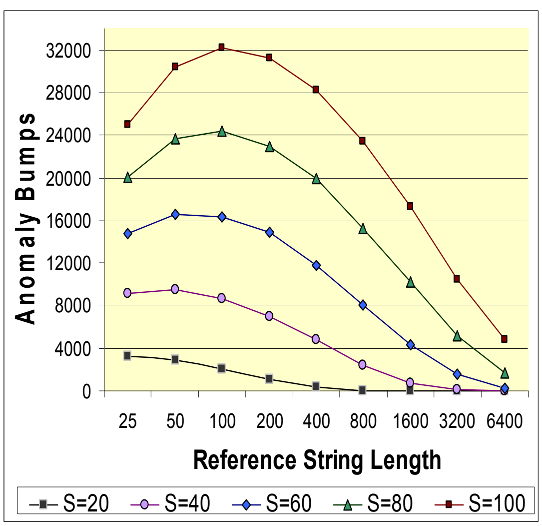

Figure 2. Random Page Anomaly Bumps per 1000 Reference Strings.

The Random Page bump pattern is similar to the FIFO results, except the bump counts are dramatically larger. Also, the maximum frequencies for each process size occur with shorter reference string lengths. Some of the reasons for these differences will be explained later in the paper.

The Random Page anomaly data used to generate Figure 2 is presented in tabular form as Table A2 in the Appendix.

2. PROBABILITY MODEL

An analysis of conditions where Belady's anomaly is likely to occur requires an appropriate probability model. Consider the memory regions for K frames vs. K+1 frames shown in Figure 3.

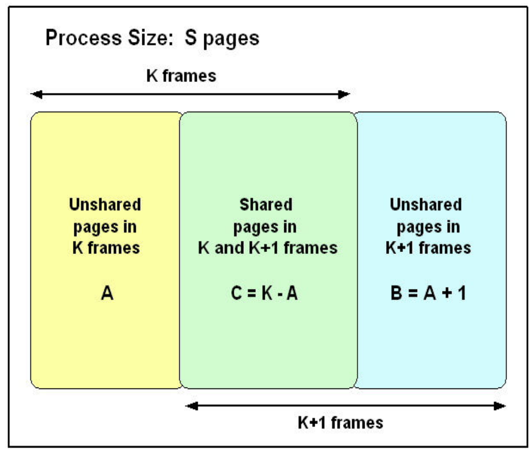

Figure 3. Memory Groups for Page References.

At any point in time, each of the S process pages will be in one of four disjoint groups:

Group 1: Unshared pages in K frames (A pages). A page reference in this group will generate a page fault for K+1 frames only.

Group 2: Unshared pages in K+1 frames (B=A+1). A page reference here will generate a page fault for K frames only.

Group 3: Shared pages in K frames and K+1 frames (C = K-A). A page reference in this group will generate a page fault for neither K nor K+1 frames.

Group 4: Not in memory (D = S–A–B–C). A page reference in Group 4 will generate page faults for both K and K+1 frames.

Belady's anomaly occurs when, for a given reference string, more page references are in Group 1 than in Group 2. This leads to more page faults for K+1 frames than K frames. Page references in Group 3 and Group 4 have no effect on the difference.

For a random page reference, probabilities can be defined for each of the four groups in which the page might be located. Define the random variable X with the probability distribution as shown in Table 1. This random variable represents the excess of page faults for K+1 frames over K frames for a *single* page reference.

Table 1. Anomaly Random Variable for one page reference.

Group	X	P[X]	Page Fault
1	1	A/S	K+1 frames
2	-1	(A+1)/S	K frames
3	0	C/S	neither
4	0	D/S	both

The probability of an anomaly depends on the number of unshared pages A relative to the process size S, and the value of A compared to A+1. If page faults for a reference string of length L are tracked using $Y = \Sigma X$, Belady's anomaly will occur when there are more page faults for K+1 frames (X = 1 values) than for K frames (X = -1 values). In this case, the random variable Y will be positive. Thus, we are interested in the probability $P[Y > 0]$, given process size S, reference string length L, and unshared pages A in K frames.

If we assume (incorrectly) that the value of A is constant, the *expected value* of Y for a random reference string of length L is:

$$E(Y) = L*E(X) = L(-1/S) = -L/S$$

The expected values of X and Y are negative, because page faults are less likely when more memory frames are available. The longer the reference string, the greater the magnitude of the expected value of Y.

Under the same assumptions, the *variance* of Y is:

$$Var(Y) = L*Var(X) = L[(2A+1)S - 1)]/S^2$$

We define the *standard score* Z_0 for Y = 0 as:

$$Z_0 = (0 - E(Y))/SDev(Y)$$
$$= Sqrt\{L/[(2A+1)S - 1]\}$$

where SDev(Y) denotes the *standard deviation* of Y. Since $P[Y > 0] = P[Z > Z_0]$, the formula for Z_0 indicates the probability that Belady's anomaly will occur depends on the following factors:

1. A longer reference string length L *decreases* the likelihood of an anomaly (larger Z_0).
2. A larger process size S *increases* the likelihood of an anomaly (smaller Z_0).
3. A greater number of unshared pages A *increases* the likelihood of an anomaly (smaller Z_0).

Additional considerations that affect Belady's anomaly include:

4. The value of A depends on S and K. The pattern of this relationship and its effect on Belady's anomaly are not obvious.
5. A larger S means that there are more frame pairs K vs. K+1 in which an anomaly can appear. This increases the likelihood of an anomaly.
6. The value of A is *not constant*. Initially, the value of A is 0 while the K frames and K+1 frames are being filled. Additional page references gradually increase the value of A. Thus, Belady's anomaly is unlikely for short reference strings.
7. The change in A depends on the page replacement algorithm. With a stack algorithm, the pages in K frames are a subset of the pages in K+1 frames. In this case, A is always 0, so Belady's anomaly never occurs. The number of unshared pages increases for FIFO and Random Page, but at different rates and with different maximum values.

3. METHODOLOGY

Computer simulation was performed using a program written specifically for this research. We focused on four conditions that affect the occurrence of Belady's anomaly:

1. Page replacement algorithm.
2. Process size in pages.
3. Reference string length.
4. Memory frames allocated to the process.

In the simulation runs, the process size ranged from 20 to 100 pages in increments of 20. The length of the reference string varied from 25 to 6400 page references, with each value being twice the preceding value. Two page replacement algorithms were examined, FIFO and Random Page.

In each simulation run, one page replacement algorithm, one process size, and one reference string length were selected. Then 1000 random reference strings were generated using a uniform random number generator. For each reference string, the number of page faults was computed for each memory allocation from 1 frame up to the process size.

For each algorithm and memory frame level, frames were "pre-filled" before any reference string page faults were counted. Specifically, K memory frames were filled with page numbers K (first-out) through 1 (last-in). Similarly, K+1 frames were filled with page numbers K+1 (first-out) through 1 (last-in). These initial allocations insured that the pages in K frames were a subset of the pages in K+1 frames, and that the pages were in the same order in the two queues (with page 1 being the most recent arrival). This initial "pre-fill" method was necessary for FIFO comparisons, although irrelevant for Random Page.

Our computer simulation calculated the number of shared pages C and unshared pages A for each pair of frame levels (K vs. K+1) under each combination of design conditions. It was expected that A would be larger (and C smaller) for the Random Page algorithm than for FIFO.

4. BELADY'S ANOMALY: FIFO

The number of unshared pages A for K frames depends on the page replacement algorithm, the process size S, the reference string length L, and the allocated frames level K. In this section, we examine the value of A for the FIFO algorithm. It is difficult to visualize the simultaneous effects of three variables on A, so we initially fixed the number of allocated frames (as a percent of process size). This allowed us to focus on how the number of unshared pages depends on the process size and string length. Later, we will explain how equilibrium values for A relate to S and K.

Unshared Pages and Reference Strings

Figure 4 describes for FIFO how A varies as the reference string length L increases up to 6400 for process sizes S between 20 and 100, when the number of allocated frames K is 80% of S. We express K as a percent of S so that the K values for different process sizes are comparable. That is, 80% represents K = 16 frames for 20 pages, K = 32 frames for 40 pages, etc.

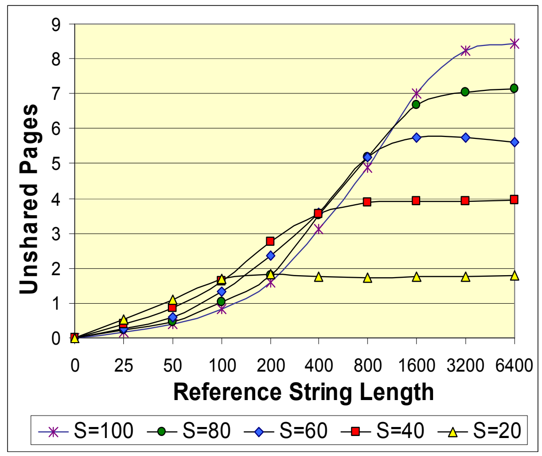

Figure 4. Unshared Pages: FIFO. By process size S and reference string length L. Frames K = 80% of S.

Under FIFO, the initial value of A for length L = 0 is always 0 because of how frames are pre-filled in the simulation. Observe that for L = 6400, the slope is approximately 0 for each process size. As the reference string length increases, the number of unshared pages approaches an equilibrium value that depends on the process size (and the number of frames). The term "equilibrium" value does not mean that A remains constant. Probabilities are involved with each page reference, so A will continue to vary randomly, but around a constant average value.

The largest value for A in Figure 4 is 8.44 unshared pages for process size S = 100. This equilibrium value is reached when L = 6400. By comparison, the maximum value for A when S = 20 is 1.77 unshared pages, which is attained when L = 200. A small process size reaches its equilibrium value sooner, since there are fewer initial pages to "flush out".

In Figure 4, we presented data for K = 80% of the process size. Different frame levels show similar patterns, but with smaller equilibrium A values. In the FIFO part of this study, the largest value of A (= 8.44) occurs when S = 100, L = 6400, and K = 80 (80% of S).

Unshared Pages and Allocated Frames

We now examine for FIFO how the number of unshared pages A depends on process size S and allocated frames K (expressed as a percent of S). To remove the effect of reference string length, we restrict our discussion to equilibrium values of A. Figure 5 summarizes the unshared pages A by process size S (20 to 100) and allocated frames K (10% to 90% of S).

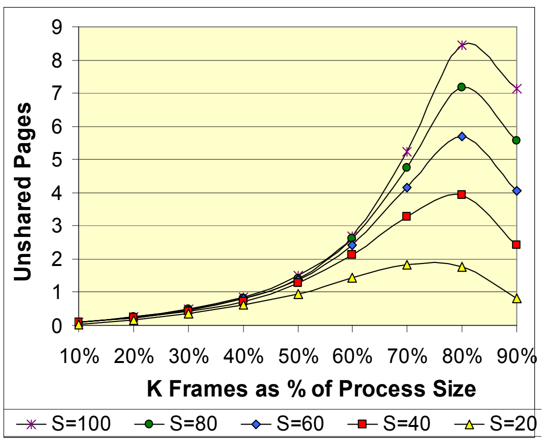

Figure 5. Unshared Pages A and Allocated Frames: FIFO. By process size, using equilibrium A values.

The FIFO pattern for equilibrium A values has some interesting features.

1. When K is 40% or less of S, all A values are below 1.0.
2. For a fixed process size S, as K grows from 10% to 90%, the value of A increases to a maximum and then starts to decrease.
3. For fixed S, the maximum value of A occurs when K is 70% (for S = 20) or 80% (for S > 20).
4. The equilibrium value of A increases as S grows larger.
5. The largest value of A in Table 3 is 8.44 unshared pages.

Very few unshared pages occur under FIFO when the allocated frame level is below 60% of S. Unshared pages are most likely to occur when S is greater than 20 and K is between 70% and 90%.

Unshared Pages and Anomalies

Previous sections have described how the number of unshared pages A depends on process size S, reference string length L and allocated frame level K. We now examine the relationship between A and the number of anomaly bumps.

Table 2 lists, for each process size S, the maximum number of FIFO anomaly bumps. These values are taken from the data presented in Figure 1. As S becomes larger, the maximum anomaly bump count consistently increases.

Table 2 also presents, for each process size S, the equilibrium number of unshared pages A when the allocated frame level K is 80% of S. The 80% frame level was chosen because this level exhibits the largest A values for FIFO. As for anomaly bumps, when S increases, the value of A becomes consistently larger.

Table 2. Anomaly Summary: FIFO. Max anomaly bumps vs. unshared pages (frames K = 80% of S)

Process Size S	Max Bumps	Unshared Pages A	A/S (%)
20	60	1.77	8.85
40	207	3.92	9.80
60	399	5.69	9.48
80	585	7.15	8.94
100	869	8.44	8.44

However, for each process size, the maximum number of anomalies occurs before A reaches its equilibrium value. Two competing effects are at work here. Anomalies increase as A becomes larger, but anomalies decrease when L grows longer. The highest anomaly bump frequency involves a tradeoff of these two effects.

Table 2 includes the ratio of unshared pages A to process size S, stated as a percentage. In our probability model defined earlier, the variance of the random variable X (and Y) depends on A/S. For FIFO, this ratio does not vary much over the range of process sizes in the study. When S is 20, the ratio A/S is slightly below 9.00%. For S values from 40 to 100, A/S decreases gradually from 9.80% to 8.44%.

The anomaly bump counts in Table 2 are the sum of anomalies across all frame levels, whereas unshared page values apply only to the 80% frame level. We can look more closely at the effect of frame level by "zooming in" on the conditions that led to the anomaly bump count of 869. In this case, S is 100, and L is 1600. The 869 bumps occurred across many frame pairs K and K+1, where K ranged from 1 to 99, and each pair had its own value of A. Instead of reporting results for all 99 frame pairs, we focus on K values 10, 20, ... 90.

The pattern of anomaly bumps and unshared pages for these nine frame levels is shown in Figure 6. Figure 6 is a line graph with two vertical axis variables, each with a separate scale. The number of unshared pages A is on the left scale, and the anomaly bump counts are on the right scale.

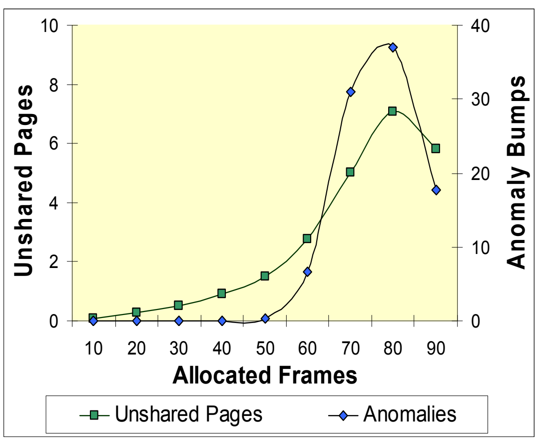

Figure 6. Unshared Pages and Anomalies: FIFO. Process size = 100 and reference string length = 1600.

Because of the randomness in the simulation, the values in Figure 6 are "smoothed." For each selected frame level K, we averaged the three A values and the three anomaly bump counts for K-1, K, and K+1 frames. Note the similarity of the pattern of each vertical variable with frame count K. For FIFO, the larger anomaly bump counts and the larger A values fall over a fairly narrow range of frame levels, primarily between 70 and 90.

5. BELADY'S ANOMALY: RANDOM PAGE

In this section, we examine relationships between unshared pages A, process size S, reference string length L, and allocated frames K for the Random Page algorithm. As with FIFO, we initially fix K (as a percent of S) and focus on how A depends on S and L. After determining equilibrium values for A, we observe the pattern of these maximum A values with S and K.

Unshared Pages and Reference Strings

Figure 7 describes how A varies as the reference string length L increases up to 6400 for process sizes S between 20 and 100, when the number of allocated frames K is 60% of S. These results are for the Random Page algorithm.

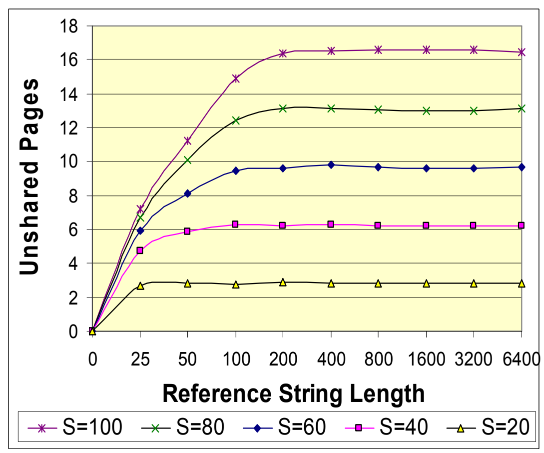

Figure 7. Unshared Pages: Random Page. By process size S and reference string length L. Frames K = 60% of S.

In this case, 60% represents K = 12 frames for 20 pages, K = 24 frames for 40 pages, etc. The initial value of A for length L = 0 is 0 because of how frames are pre-filled in the simulation. As L increases beyond 100, the number of unshared pages approaches an equilibrium value that depends on the process size. For L = 200, the slope of the line is approximately 0 for each process size.

The largest value for A in Figure 7 is 16.51 unshared pages for a process size of S = 100. This equilibrium value is reached when length L = 200. By comparison, the maximum value for A when S = 20 is 2.82 unshared pages, which is attained when L =

50. With the Random Page algorithm, the initial pages are "flushed out" more rapidly than with FIFO.

In Figure 7, we selected K = 60% of S, since this yields the largest equilibrium values for A. In the Random Page part of this study, the largest value of A (= 16.51) occurs when S = 100, L = 200, and K = 60 (60% of S).

Unshared Pages and Allocated Frames

We now consider for Random Page how the number of unshared pages A depends on process size S and allocated frames K (expressed as a percent of S). Figure 8 summarizes the equilibrium values of A by process size S (20 to 100) and allocated frames K (10% to 90% of S).

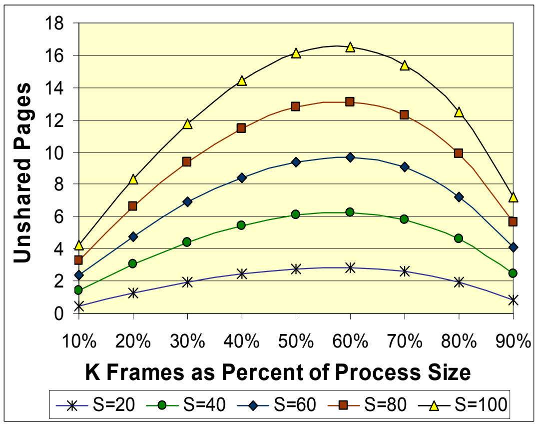

Figure 8. Unshared Pages A and Allocated Frames: Random Page. By process size, using equilibrium A values.

The Random Page pattern for equilibrium A values differs somewhat from FIFO.

1. The only A value less than 1.0 occurs when S is 20 and K is 10% of S.
2. For each fixed process size S, as K grows from 10% to 90%, the value of A increases to a maximum, and then starts to decrease.
3. For each fixed S, the maximum value of A occurs when K is 60% of S.
4. Larger process sizes have larger equilibrium A values.
5. The largest value of A in Figure 8 is 16.51 unshared pages.
6. Equilibrium A values for Random Page are often twice the size of corresponding A values for FIFO.

Relatively few unshared pages occur under Random Page when the allocated frame level is near 0% or is almost 100%. More unshared pages commonly occur when S is greater than 20 and K is between 30% and 80%. Under Random Page, large unshared page counts occur over a wider range of frame levels than for FIFO.

Unshared Pages and Anomalies

We now describe the relationship between unshared pages A and the number of anomaly bumps for the Random Page algorithm. Table 3 lists, for each process size S, the maximum number of anomaly bumps. These values are taken from the data presented in Figure 2. As S becomes larger, the maximum anomaly bump count grows substantially.

Table 3. Anomaly Summary: Random Page. Max anomaly bumps vs. unshared pages (frames K = 60% of S).

Process Size S	Max Bumps	Unshared Pages A	A/S (%)
20	3236	2.82	14.10
40	9547	6.23	15.58
60	16638	9.66	16.10
80	24400	13.08	16.35
100	32152	16.51	16.51

Table 3 also shows, for each process size S, the equilibrium unshared pages A when the allocated frame level K is 60% of S. The 60% frame level was chosen because this level exhibits the largest A values for Random Page. When S increases, the value of A becomes larger, but requires slightly longer reference strings.

For Random Page, the reference string lengths for maximum anomaly bumps are fairly short, and the corresponding lengths needed for A to reach equilibrium are not much longer. Even so, the maximum number of anomalies occurs before A reaches its equilibrium value. As with FIFO, anomalies increase as A becomes larger, but decrease when L grows longer.

Table 3 includes the ratio of unshared pages A to process size S, stated as a percentage. In our probability model defined earlier, the variance of the random variable X (and Y) depends on A/S. For Random Page, this ratio increases gradually from 14.10% (S = 20) to 16.51% (S = 100). The Random Page A and A/S values are 1.5 to 2 times larger than the FIFO values.

The anomaly bump counts in Table 3 are the sum of anomalies across all frame levels, whereas unshared page values apply only to the 60% frame level. We can look more closely at the effect of frame level by detailing parts of the total anomaly bump count of 32152. In this case, S is 100, and L is 100. The 32152 bumps occurred across most frame pairs K and K+1, where K ranged from 1 to 99, with each pair having its own value of A.

The pattern of anomaly bumps and unshared pages for the nine specific K values 10, 20, ... 90 is displayed in Figure 9.

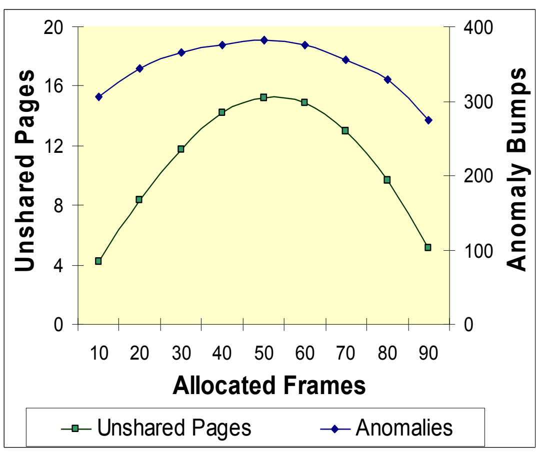

Figure 9. Unshared Pages and Anomalies: Random Page. Process size = 100 and reference string length = 100.

Like Figure 6, Figure 9 is a line graph with two vertical axis variables. The number of unshared pages A is on the left scale, and the anomaly bump counts are on the right scale. Because of the randomness in the simulation, the values in Figure 9 are "smoothed" in the same manner as Figure 6.

The two vertical variables have similar patterns with respect to allocated frames. For Random Page, large anomaly bump counts and relatively large A values extend over a wide range of frame levels between 10 and 90.

6. SUMMARY AND CONCLUSIONS

In this study we provided a probability model to explain the occurrence of Belady's anomaly in terms of unshared pages for K vs. K+1 memory frames. We used computer simulation to estimate the number of unshared pages under four conditions: (1) page replacement algorithm, (2) process size, (3) reference string length, and (4) allocated frames. We then related the number of unshared pages to anomaly bump counts obtained in an earlier study.

For the FIFO and Random Page algorithms, we found that for each process size S and frame allocation level K, the number of unshared pages A increased from 0 up to an equilibrium value as the reference string became longer. As S increased, the A values became larger, but required longer reference strings to reach equilibrium. For Random Page, maximum A values were attained when the reference string length was 400 or less. FIFO required up to 6400 page references for A values to stabilize.

In some cases, the equilibrium values of A were almost twice as high for the Random Page algorithm vs. FIFO. The maximum values of A occurred when K was 80% of S for FIFO, but when K was 60% of S for Random Page. Large A values and anomaly bump counts occurred over a narrow range of frame levels (70% to 90% of S) for FIFO, but over a wider range (10% through 90% of S) for Random Page.

Belady's anomaly bumps are more frequent for large A values, especially when the large A values occur over a wide range of frame levels. Since long references strings decrease the probability of an anomaly bump, anomalies are more likely when equilibrium A values are reached quickly. These unshared page features combine to produce much higher Belady's anomaly occurrence rates for Random Page than for FIFO.

Future Research

We are continuing our research on Belady's anomaly. Alternative probability models such as the Binomial distribution are being considered. Statistical goodness-of-fit tests to evaluate the suitability of the models are being performed. The Random Page algorithm justifies many of the assumptions required by probability models and statistical tests. This is less true for the FIFO algorithm. One of our goals is to gain a better understanding of why the FIFO algorithm behaves in its unique ways.

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APPENDIX

Table A1: Total Anomaly Bumps – FIFO

Number of page fault bumps per 1000 reference strings.

Some reference strings exhibit more than one bump.

Maximum frequency for each process size is shown in **bold**.

Process Size	Reference String Length								
	25	50	100	200	400	800	1600	3200	6400
20	0	14	57	**60**	16	3	0	0	0
40	0	8	49	166	**207**	144	31	2	1
60	0	3	23	147	334	**399**	233	59	6
80	0	0	16	112	374	**585**	565	232	51
100	0	0	2	80	357	693	**869**	533	161

Table A2: Total Anomaly Bumps – Random Page

Number of page fault bumps per 1000 reference strings.

Many reference strings exhibit multiple bumps.

Maximum frequency for each process size is shown in **bold**.

Process Size	Reference String Length								
	25	50	100	200	400	800	1600	3200	6400
20	**3236**	2857	2016	1027	331	45	2	0	0
40	9184	**9547**	8707	7020	4856	2439	780	103	2
60	14770	**16638**	16391	14861	11809	8070	4268	1549	271
80	20054	23615	**24400**	22929	19965	15269	10213	5131	1660
100	24936	30351	**32152**	31229	28204	23427	17294	10416	4829